For Supervisor's use only

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90635

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Calculus, 2004

90635 Differentiate and use derivatives to solve problems

Credits: Six
9.30 am Tuesday 23 November 2004

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.
Show any derivatives that you need to find when solving the problems.

If you need more space for any answer, use the pages provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement Criteria	*For Assessor's use only*	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Differentiate functions and use differentiation to solve problems.	Demonstrate knowledge of concepts and techniques of differentiation.	Solve problem(s) involving a combination of differentiation techniques.
	Solve differentiation problems.	
Overall Level of Performance (all criteria within a column are met)		

You are advised to spend 50 minutes answering the questions in this booklet.

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Show **ALL** working.

QUESTION ONE

Differentiate the following functions. You do not need to simplify your answers.

(a) $y = (3x^2 - 7x)^5$

(b) $y = \ln(5x^4 + 1)$

(c) $y = \sec 7x$

QUESTION TWO

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The total cost of producing calculators during each production run for a certain company

is given by $$C = \frac{6\ 000\ 000}{x} + 0.3x,\ \ x \neq 0$$

where C is the total cost in dollars
and x is the number of calculators manufactured during each production run.

Determine the number of calculators that should be manufactured during each production run so that the total cost is minimised. [You may assume that $\frac{d^2C}{dx^2} > 0$.]

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QUESTION THREE

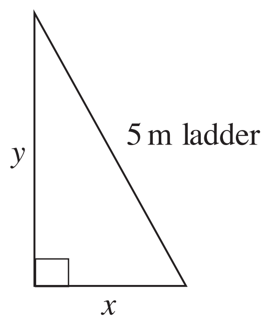
Find the coordinates of the turning points of the curve $f(x) = 8x^2 - x^4$ and determine their nature.

QUESTION FOUR

A 5 metre ladder is placed against a wall.

The top of the ladder is sliding down the wall at 0.5 metres per second.

At what rate is the bottom of the ladder moving away from the wall when the bottom of the ladder is 3 metres from the wall?



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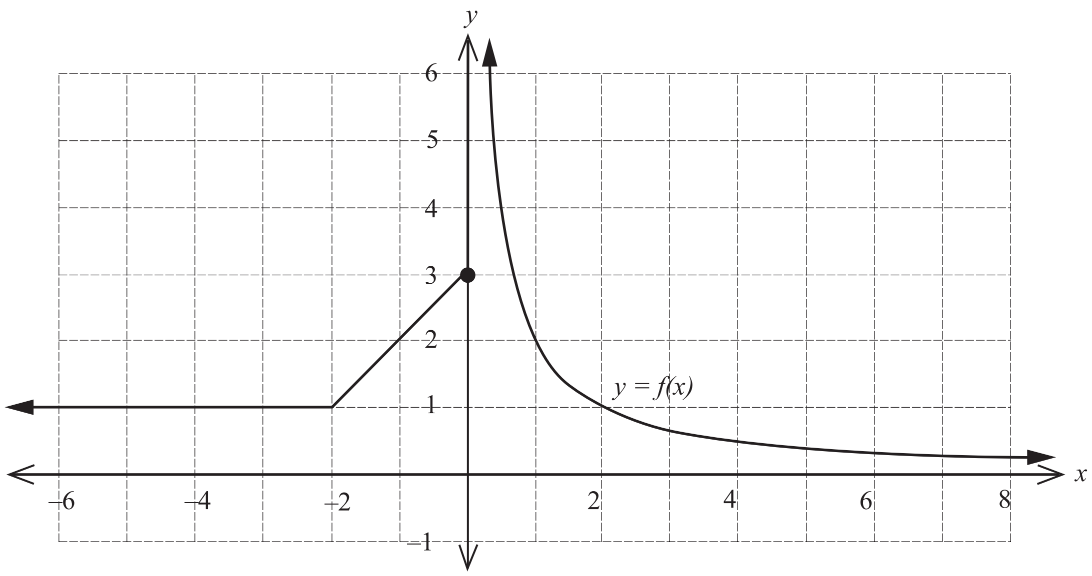
QUESTION FIVE

Differentiate the function: $y = \sqrt{3x-2}\,\sin x$. You do not need to simplify your answer.

QUESTION SIX



From the graph of $y = f(x)$ state:

(a) $\lim\limits_{x \to -2} f(x)$

(b) $\lim\limits_{x \to \infty} f(x)$

(c) the value(s) of x when $y = f(x)$ is not differentiable.

QUESTION SEVEN

Use implicit differentiation to find $\frac{dy}{dx}$ when $3x^2 - 4y^2 = 10$.

QUESTION EIGHT

The concentration $C(t)$ of a particular fertiliser in the soil is given by

$$C(t) = \frac{2t}{(t+3)^2}$$

where $C(t)$ is the concentration in kilograms per cubic metre
and t is the number of days after the fertiliser is applied.

How many days after the fertiliser is applied will the concentration be a maximum?
[You may assume that $C''(t) < 0$.]

QUESTION NINE

In the morning, the angle of elevation of the sun is increasing at a rate of $\frac{\pi}{540}$ radians per minute.

The height of a building is 150 metres.

Find the rate of change of the length of the building's shadow when the angle of elevation of the sun is $\frac{\pi}{6}$.

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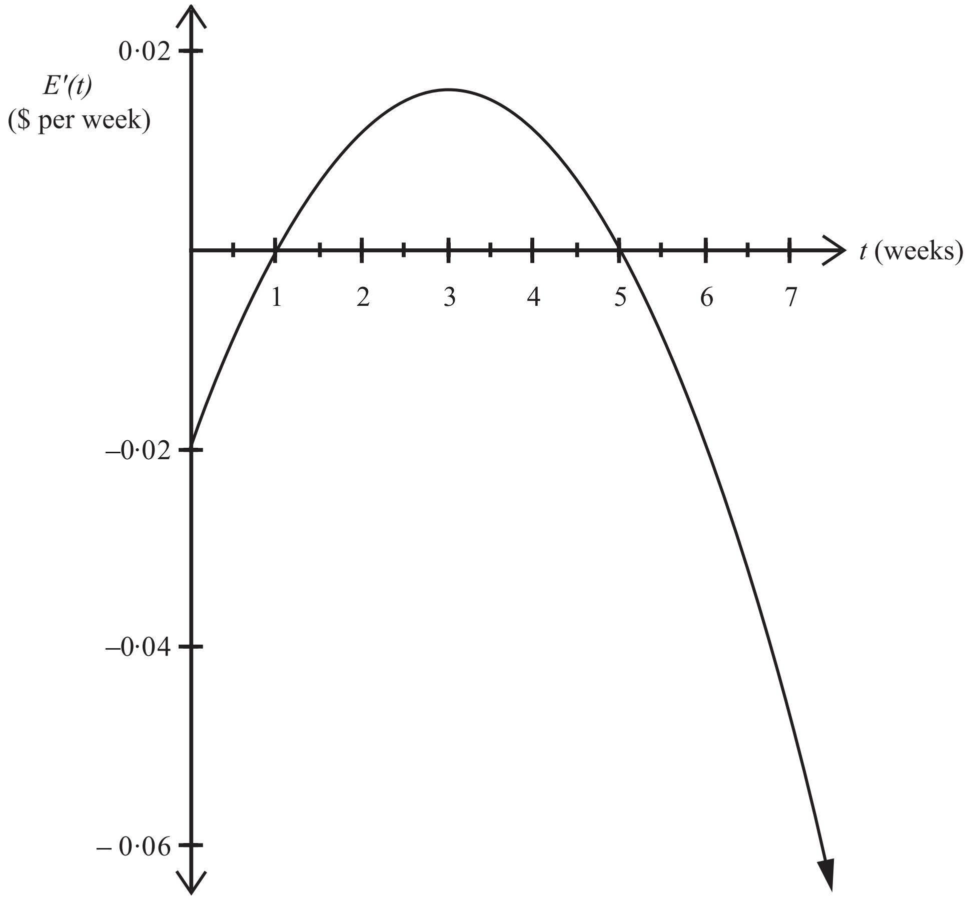
QUESTION TEN

The graph shown below approximates the rate of change of the price of apricots over a 7-week period

where $E(t)$ is the price of a kilogram of apricots in dollars
and t is the time in weeks.



(a) Describe what is happening to the price of apricots at 5 weeks.

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(b) Describe what is happening to the price of apricots at 3 weeks.

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(c) Describe what is happening to the price of apricots during the first week.

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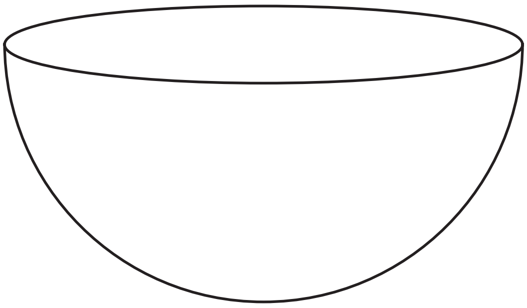
QUESTION ELEVEN

Water is poured into a hemispherical fish bowl with a radius of 20 cm.



The volume of the water in the bowl increases at a rate of 80 cm^3 per second.

(a) Find an expression for V, the volume of water in the bowl, in terms of h, the depth of water at the centre of the bowl.

(b) Hence, find the rate at which the depth of the water, h, is increasing when the depth is 5 cm.

**Extra paper for continuation of answers if required.
Clearly number the question.**

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Question number

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Clearly number the question.

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Question number